IFAC World Congress 2017 - Tutorial Session on

Cooperative and noncooperative decision making in multi-agent systems: An operator theoretic perspective

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Saturday 8 July 2017, 9:00–17:30

Topic and scope

Motivated by applications in energy and transportation networks, this tutorial will present a mathematical framework to analyse and design distributed decision making in multi-agent systems seeking convergence to cooperative or noncooperative equilibria.

By using a systems-and-control perspective, the proposed tutorial aims at achieving the following two objectives:

- build a mathematical background for the analysis and design of decision making algorithms in cooperative and noncooperative multi-agent systems, and provide a unifying framework for studying and analyzing decision making problems that arise in contemporary societal and technological challenges;
- provide targeted education and training to graduate students and young researchers, and specifically to the next generation of systems and control engineers, researchers and technology advisors.

The tutorial aims at creating a common mathematical ground for the many students and researchers in the IFAC community who will work on, or are currently working on, optimal decision making in multi-agent systems[1].

Structure and schedule

The tutorial is structured in the following four sessions.

S0. **Motivation and illustrative applications**

Introduction to decision making problems and applications to the energy and transportation domains:

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(a) Energy efficient control of district buildings.

(b) Smart grid control and optimization.

(c) Coordination and control for electric vehicle fleets.

These applications not only will serve as a motivation, but will also set a common ground to highlight the main complexity issues arising in multi-agent systems, namely, scalability due to the large scale nature of these problems, heterogeneity of the agents and privacy. A distinction between cooperative and noncooperative agent strategies will also be made depending on the considered application.

S1. **Mathematical tools from operator theory**

(a) Convex analysis, optimization and duality theory.

(b) Operator theory basics.

(c) Fixed point algorithms.

These tools constitute the theoretical backbone for the analysis and design of cooperative and noncooperative algorithms that arise in multi-agent decision making problems.

S2. **Cooperative and noncooperative decision making**

(a) Cooperative optimization: primal-based algorithms.
Topics covered are gradient and scaled projected gradient methods, Jacobi algorithm, and Gauss–Seidel algorithm. The background notions on convex optimization and fixed point algorithms provided in Session S1 will be used.

(b) Cooperative optimization: duality-based algorithms.
Topics covered are distributed dual decomposition and distributed alternating direction method of multipliers (ADMM). The background notions on duality theory and dual algorithms provided in Session S1 will be used.

(c) Noncooperative equilibrium seeking algorithms.
Topics covered are the best-response dynamics and the fictitious-play dynamics. The background notions on operator theory and fixed point theorems provided in Session S1 will be used.

The selected topics in this part provide a wide coverage of cooperative and noncooperative algorithms that arise in the multi-agent decision making. More advanced topics are addressed in Session S3.

S3. **Advanced topics and vistas**

The session will provide a glance at advanced operator theoretic solutions for cooperative and non-cooperative multi-agent decision making, and at the bridge between the two perspectives when game equilibria result in social welfare, always keeping an eye on the motivating applications in S0.

A tentative agenda illustrating the timeline for the tutorial sessions is planned as follows.

09.00: Start of the Tutorial
S0: Motivation and illustrative applications (Prandini)
S1: Mathematical tools from operator theory (Notarstefano, Grammatico)
Convex analysis, optimization and duality theory
10.30-10.45: Coffee break
Operator theory fundamentals
Fixed point algorithms
12:30-14.00: Lunch break
S2: Cooperative and noncooperative decision making (Notarstefano, Margellos, Grammatico)
Cooperative optimization: primal-based algorithms
Cooperative optimization: duality-based algorithms
15.30-15.45: Coffee break
Noncooperative equilibrium seeking
S3: Advanced topics and vistas (Margellos, Prandini)
17.30: End of the Tutorial

Intended audience and prerequisites

The intended audience consists of graduate students enrolled in a post-master, doctoral or post-doctoral research program in at least one of or across the following areas: multi-agent systems and control, optimization, computational game theory, distributed decision making.

Since the tutorial will cover some advanced topics in mathematical analysis and operations research, basic mathematical knowledge is required. Specifically, the following prerequisite knowledge is recommended for the audience:

- mathematical analysis;
- optimization theory;
- linear algebra (basic level);
- systems and control theory (basic level).

No background knowledge in operator theory or game theory is required.

Expected learning outcomes

By attending the tutorial, the participants are expected to achieve the following learning outcomes:

- strengthening knowledge in optimization;
- knowledge of the basic principles in operator theory;
- recognizing and understanding convergence issues in multi-agent systems;
- bridging operator theoretic tools with convergence problems in iterative algorithms;
- analysis of multi-agent control and optimization algorithms under an operator theoretic lens;
- familiarizing with multi-agent applications and problems of contemporary interest.